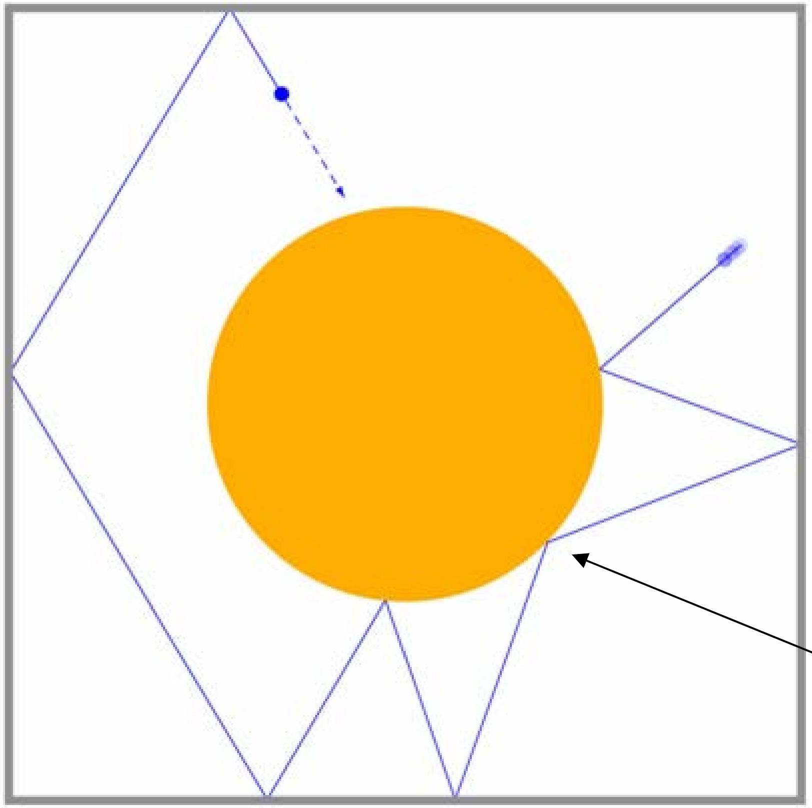
ქვანტური ბილიარდის ოპტიკური ვიზუალიზაცია

Chaotic Classical Billiards

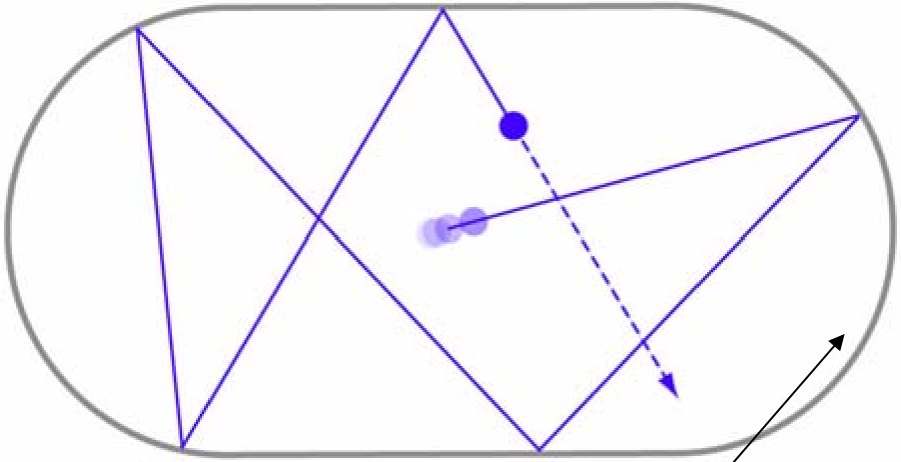
$$H(\vec{p},\vec{r}) = \frac{p^2}{2m} + V(\vec{r})$$

$$V(\vec{r}) = \begin{cases} 0, & \vec{r} \in \mathbf{S} \\ \infty, & \vec{r} \notin \mathbf{S} \end{cases}$$

Sinai Billiard



Bunimovich Stadium



Convex and Concave scatters

Quantum Billiards

$$i\frac{\partial\Psi}{\partial t} = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2}, \qquad \Psi(\vec{r},\mathrm{t})\big|_{\mathrm{s}} = 0$$

$$\Psi(\vec{r},t) = \sum_n e^{i\lambda_n t} A_n(\vec{r}); \qquad -\lambda_n A_n(x,y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) A_n(x,y)$$

$$\text{sorting } \lambda_n \text{ we define } s_n = |\lambda_{n+1} - \lambda_n|$$

Regular Billiards –
Poisson Distribution

$$R(s) = \exp[-s]$$

Chaotic Billiards –
Wigner Distribution

$$R(s) = \frac{\pi}{2} s \cdot \exp\left[-\pi s^2/4\right]$$

Discrete Quantum Billiards

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \rightarrow \Psi_{m,n+1} + \Psi_{m,n-1} + \Psi_{m+1,n} + \Psi_{m\text{-}1,n} - 4\Psi_{m,n}$$

$$i\frac{\partial \Psi_{m,n}}{\partial t} = \Psi_{m,n+1} + \Psi_{m,n-1} + \Psi_{m+1,n} + \Psi_{m\text{-}1,n} - 4\Psi_{m,n}$$

$$\Psi_{m,n} = \psi_{mn} e^{4it}$$

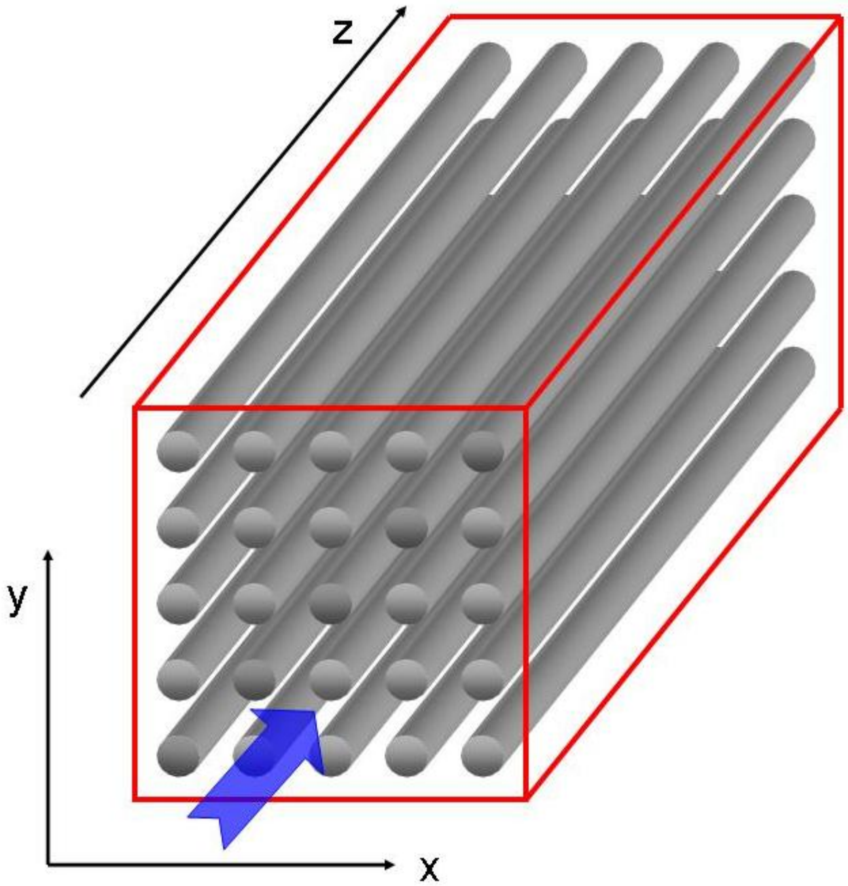
$$i\frac{\partial \psi_{m,n}}{\partial t} = \psi_{m,n+1} + \psi_{m,n-1} + \psi_{m+1,n} + \psi_{m\text{-}1,n}$$

Square Boundaries:

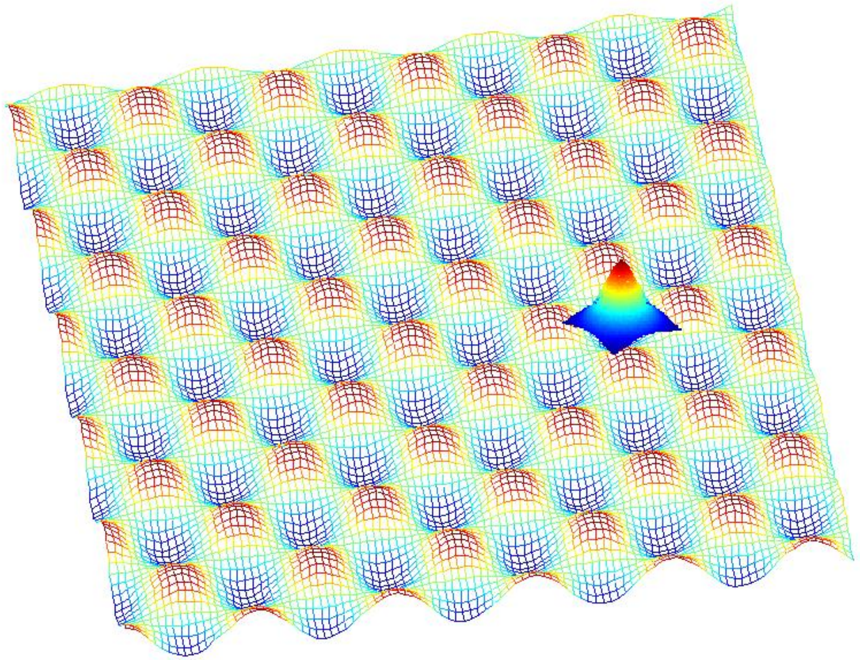
$\Psi_{m,n} = 0$ if m = 0, or m = N, or n = 0, or n > N

Realizations for Discrete Quantum Billiards

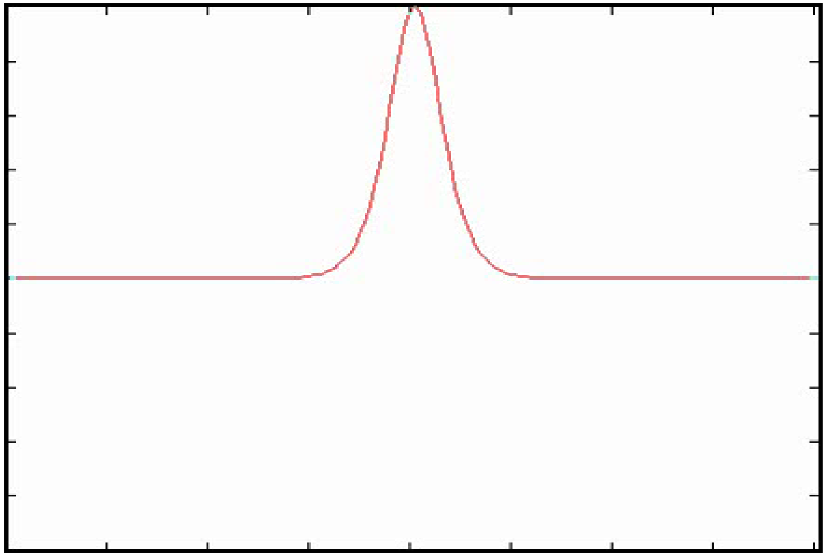
2D waveguide arrays

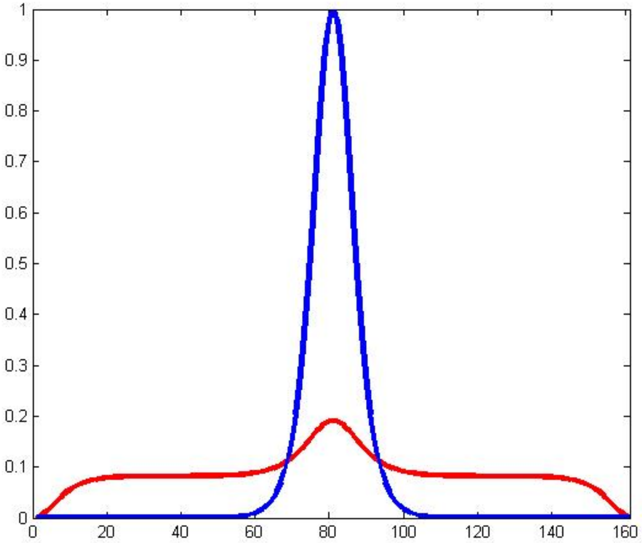


BEC in 2D optical lattices

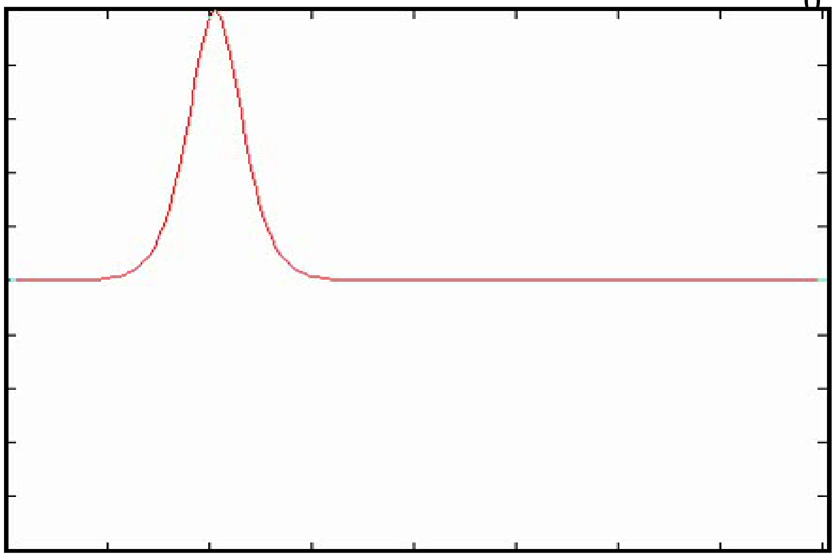


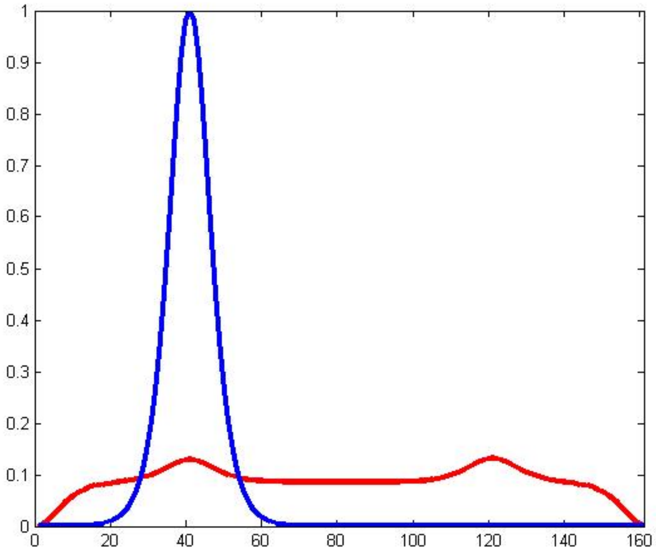
Continuous 1D Quantum Billiard



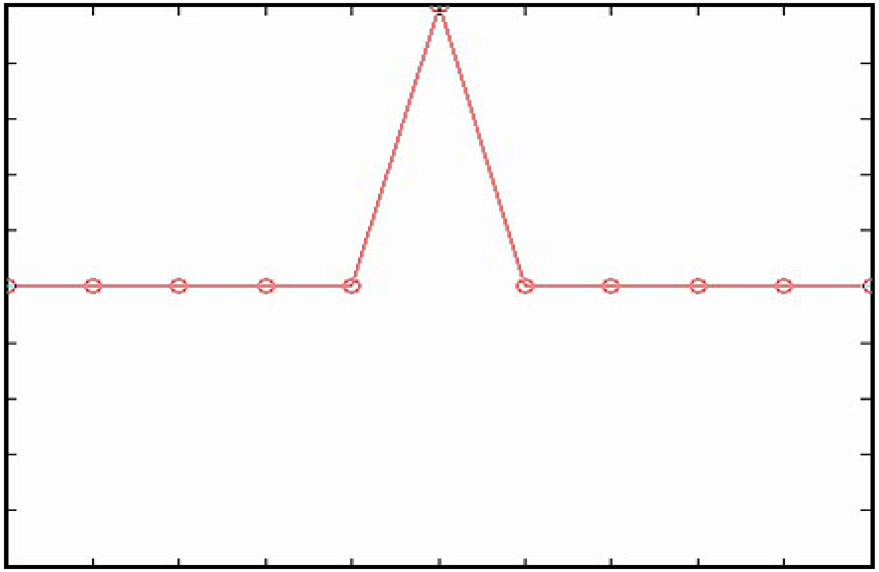


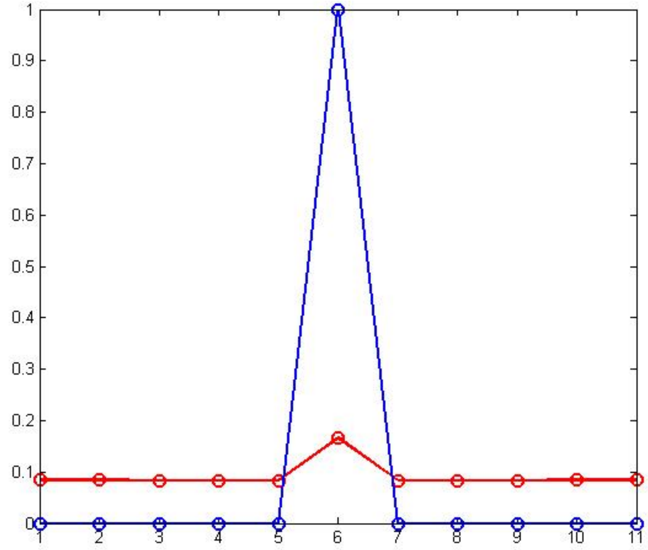
$$P(r)=\lim_{T\to\infty}\frac{1}{T}\int_0^T\left|\Psi(r,t)\right|^2 dt$$



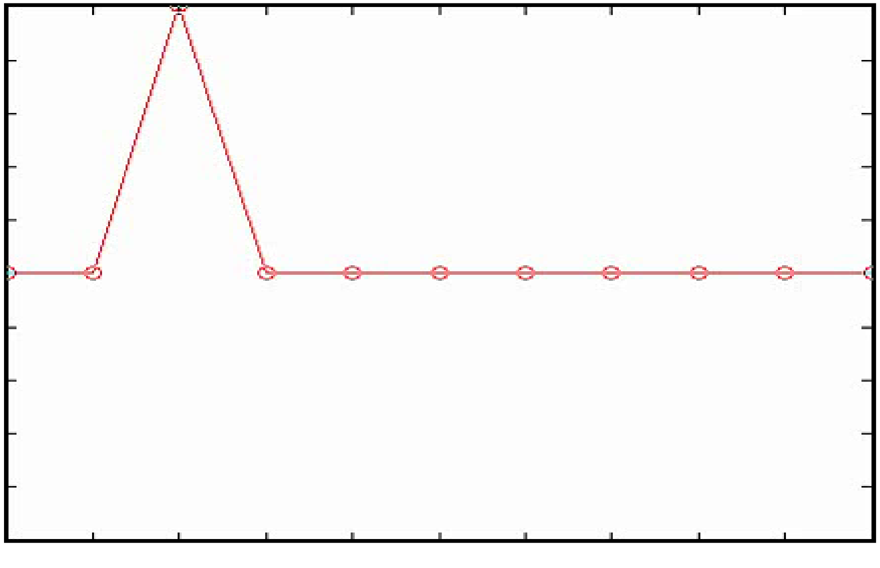


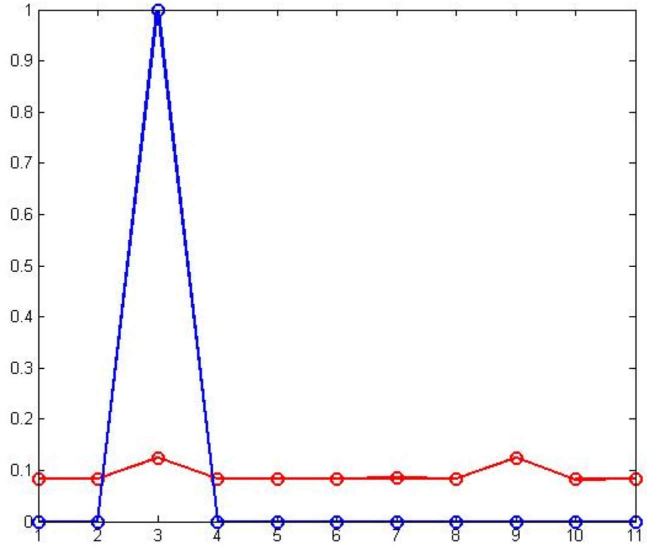
Discrete 1D Quantum Billiard





$$P(n) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left| \Psi(n,t) \right|^2 dt$$





Calculation of Probability Distribution Function (PDF)

$$P_{m,n} = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left|\psi_{m,n}\right|^2 dt$$

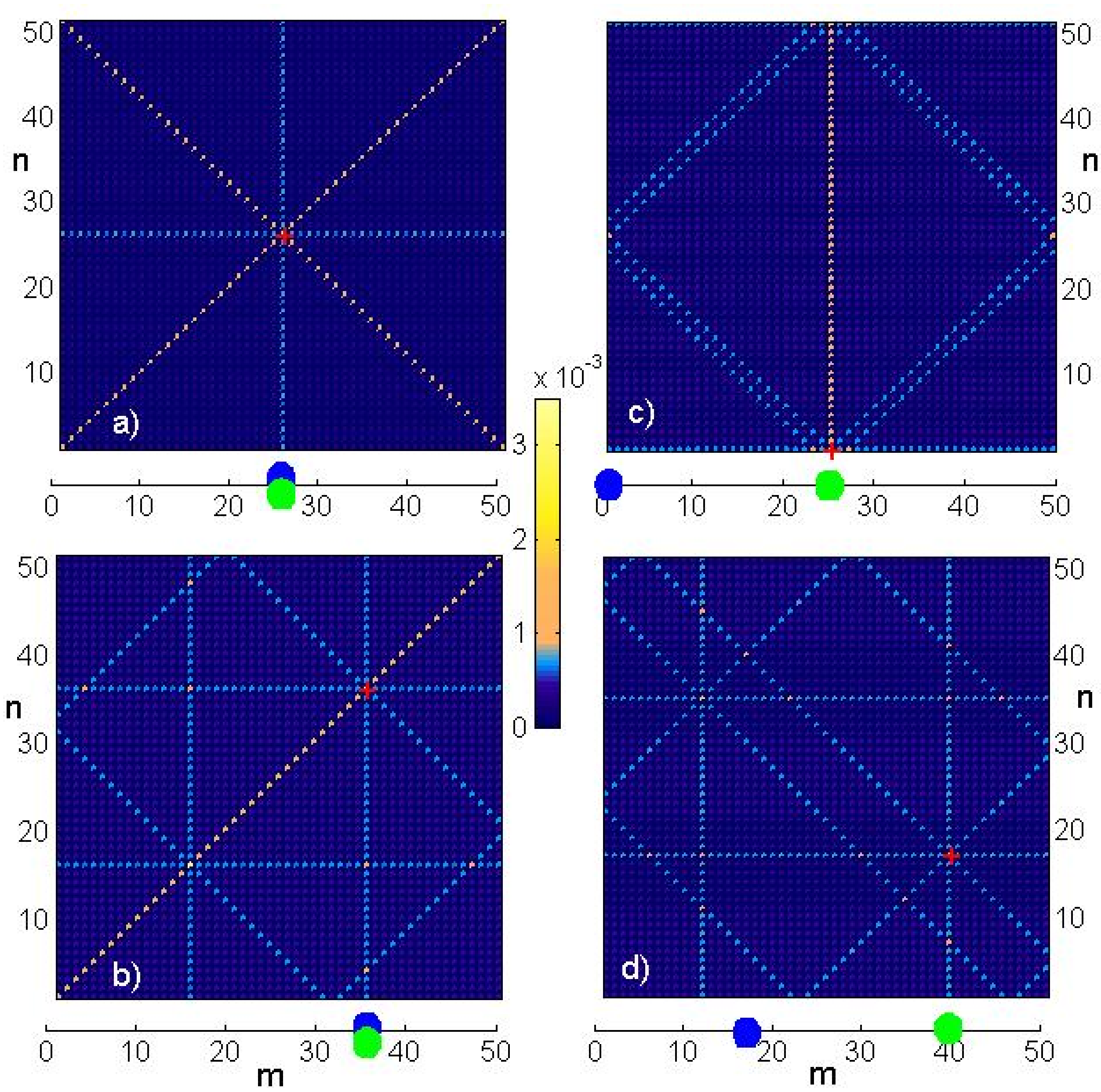
$$\left.\begin{aligned} \hat{H}\left|q\right\rangle = \lambda_q \left|q\right\rangle \quad \Rightarrow \quad & \left|\Psi(\mathrm{t})\right\rangle = \sum_q \varphi_q e^{i\lambda_q t} \left|q\right\rangle \\ & \left|\Psi(t)\right\rangle = \sum_{m,n=1}^{N} \psi_{mn}(t) \left|m,n\right\rangle \end{aligned}\right\}$$

$$\psi_{mn}(t) = \sum_q \varphi_q L_{mn}^q e^{-i\lambda_q t} \left|m,n\right\rangle \quad \text{where} \quad L_{mn}^q = \left\langle m,n \middle| q\right\rangle$$

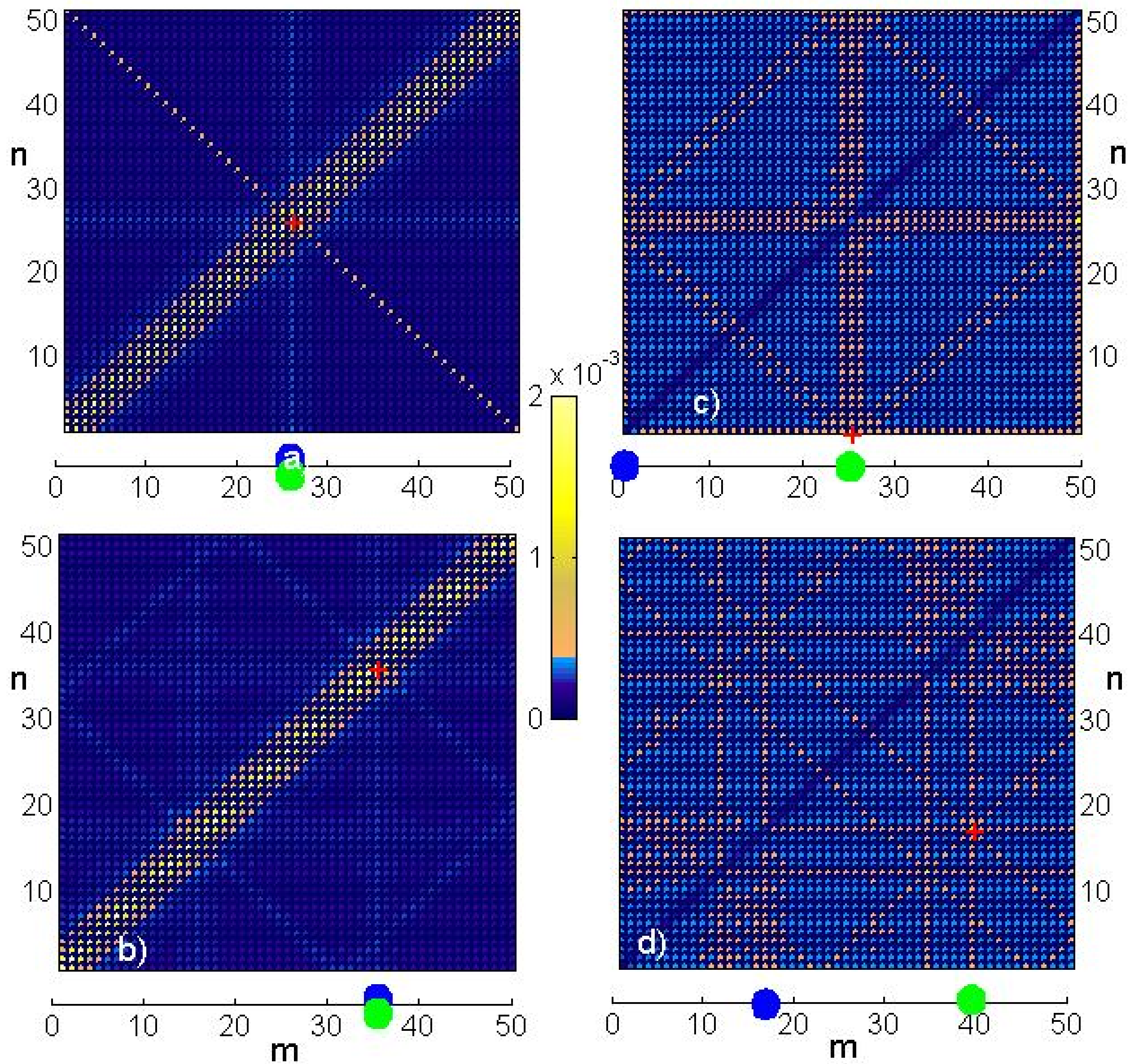
$$\varphi_q = \sum_{m,n=1}^{N} L_{mn}^q \psi_{mn}(0)$$

$$P_{mn} = \sum_{q'} \left|\varphi_{q'}\right|^2 \left(L_{mn}^{q'}\right)^2 + \sum_i \left|\sum_{q_i=1}^{r_i} \varphi_{q_i} L_{mn}^{q_i}\right|^2$$

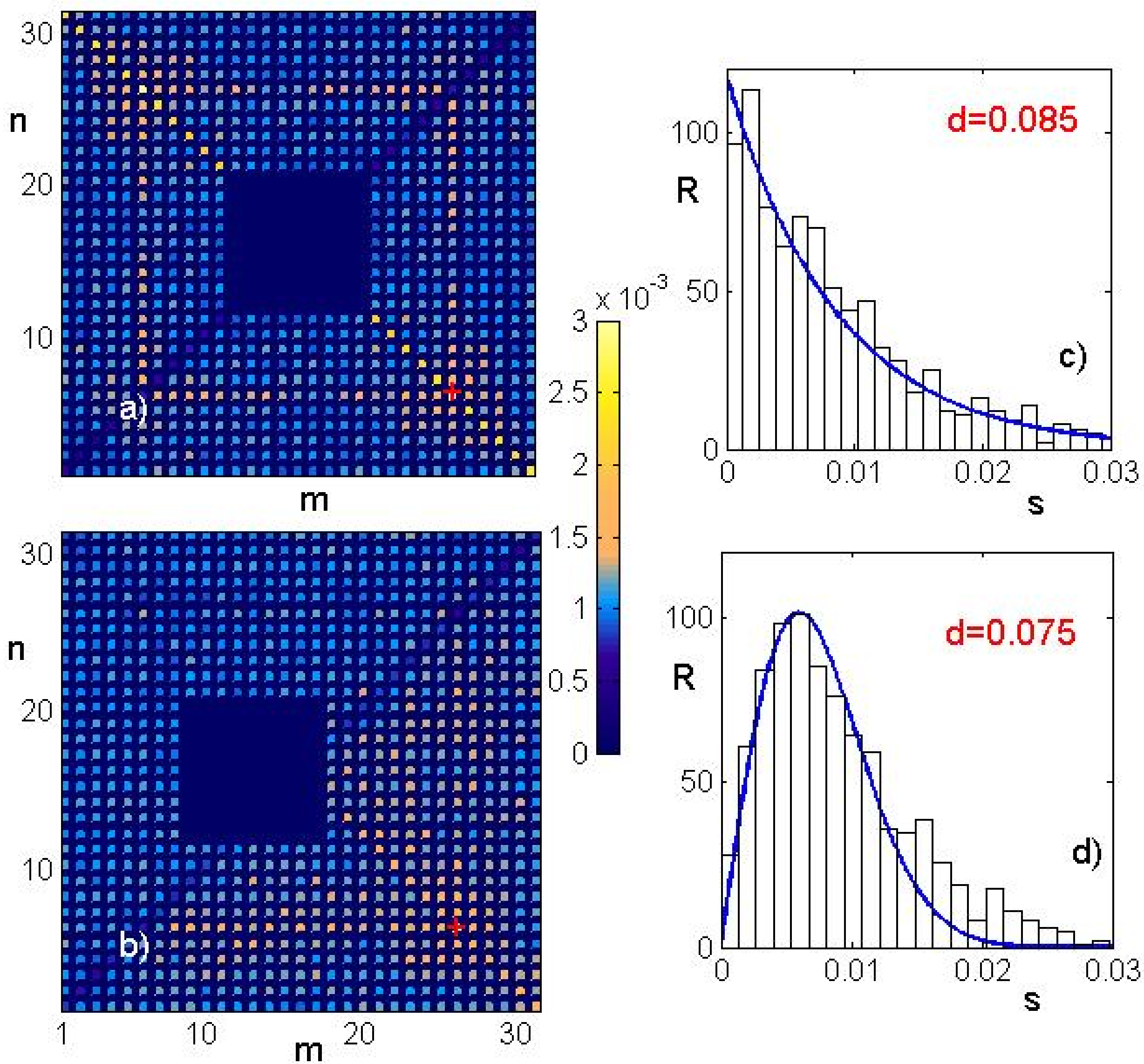
PDF for the case U=0



PDF for the case U=1



PDF for the case U=2



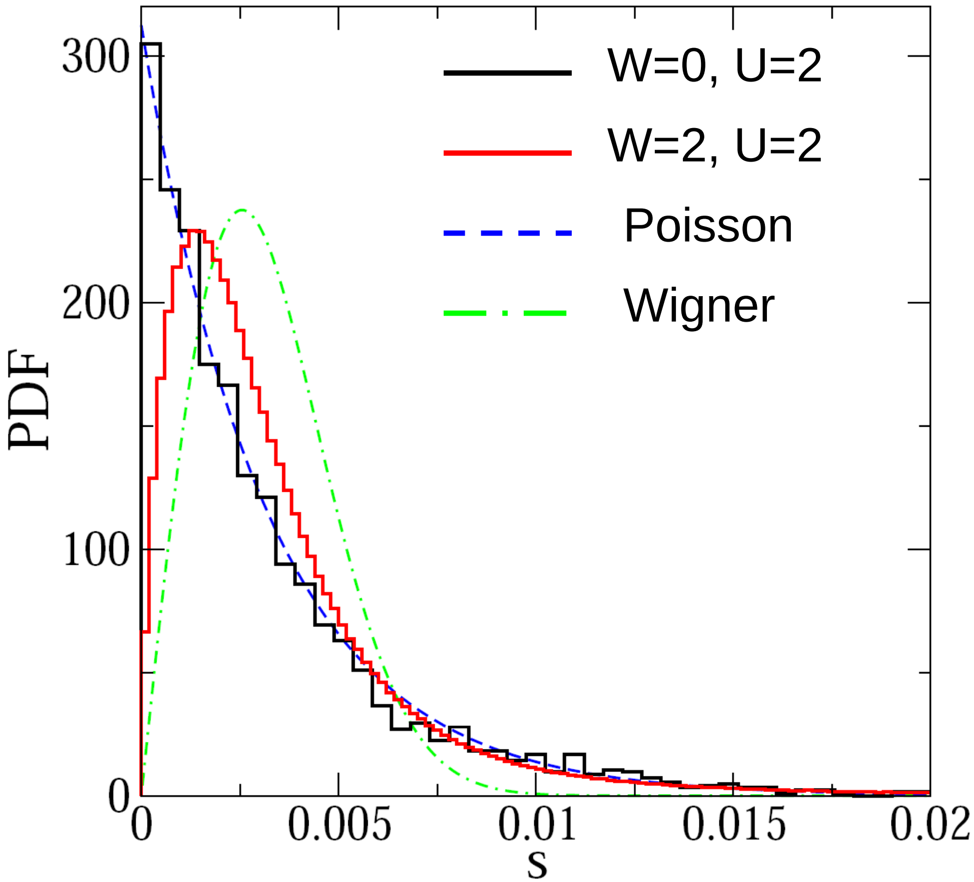
$$R(s) = \frac{1}{d}\exp\left[-\frac{s}{d}\right]$$

$$R(s) = \frac{\pi s}{2d^2}\exp\left[-\frac{\pi s^2}{4d^2}\right]$$

Disordered 2D Billiard

$$\hat{H} = \sum_{j=1}^{N-1} \left(\hat{a}_{j+1}^{+} \hat{a}_{j} + \hat{a}_{j}^{+} \hat{a}_{j+1} + \hat{b}_{j+1}^{+} \hat{b}_{j} + \hat{b}_{j}^{+} \hat{b}_{j+1} \right) + U \sum_{j=1}^{N} \hat{a}_{j}^{+} \hat{b}_{j}^{+} \hat{a}_{j} \hat{b}_{j} + \sum_{j=1}^{N} \varepsilon_j \left(\hat{a}_{j}^{+} \hat{a}_{j} + \hat{b}_{j}^{+} \hat{b}_{j} \right)$$

$$i\partial_t \Psi_{mn} = \left(W_{mn} + U\delta_{mn} \right) + \sum_{m',n'=1}^{N} R_{mn}^{m'n'} \Psi_{m'n'}$$



Correlated Disorder

$$W_{mn} = \varepsilon_m + \varepsilon_n$$

$$\varepsilon_j \in \left[-W/2, W/2 \right]$$

Uncorrelated Disorder

Each W_{mn} is a sum of two random numbers

PDF for the case W=2, U=2